# Diffusion of Drag Reducing Polymers in a Turbulent Boundary Layer

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**The diffusion of diluted drag reducing polymers and the effect of the diffusing polymers on the development of the boundary layer are analyzed. The analysis suggests that the diffusion rate is reduced together with the drag. However, in most practical situations the reduction in the diffusion rate is small. Polymers ejected near the wall will diffuse over the entire turbulent boundary layer within a distance of the order of 100 boundary-layer thicknesses.**

## Introduction

THE efficiency of drag reduction by ejection of polymers into external boundary layers is determined to a large extent by the diffusion of the ejected polymers from the wall region. At large distances from a source of a diffusing tracer, the diffusion is determined by the growth of the boundary layer, and the concentrations can be calculated by an equation proposed by Poreh and Cermak.[1] Granville[2] has used this equation to study the development of a boundary layer with polymers. The initial conditions which appear in Granville's solution are determined, however, by the diffusion closer to the source where the diffusion boundary layer is submerged within the momentum boundary layer.

The primary purpose of this work is to examine the diffusion of polymers within a developing turbulent boundary layer. The lack of detailed theories for both diffusion in turbulent shear flows and drag reduction makes it impossible to analyze rigorously the mutual interaction of the diffusing polymers and the shear flow. Similarity laws and semiempirical methods describing the diffusion pattern of an ideal tracer, and drag reduction in homogeneous solutions may be used, however, to estimate the effect of the drag reducing properties of dilute polymer solutions on the diffusion rate and to calculate the drag reduction in cases of polymer injections. Such an estimate for a zero pressure gradient boundary layer is presented herein.

## Preliminary Considerations

Although the mechanism of drag reduction is not fully understood, it is apparent from almost all measurements in boundary-layer flows that the drag reduction is associated with an increased thickness of the viscous sublayer, the remaining portion of the boundary layer remaining essentially Newtonian in nature.[3,4] Measurements have also indicated that drag reduction occurs only when the shear stress at the wall exceeds a critical value. These observations have led Meyer[5] to suggest that the mean velocity $u$ in pipe flows of dilute polymer solutions be described by the following equations:

$$u/V^* = A \log(zV^*/\nu) + B_1 + \Delta B \quad (1)$$

where the logarithms are natural

$$\Delta B = \alpha \log(V^*/V_{\text{crit}}^*) \quad (2)$$

$z$ is the distance from the wall, $V^*$ is the shear velocity, $V_{\text{crit}}^*$ is the shear velocity at the onset of drag reduction, $A$, $B_1$ are the constants used for Newtonian fluids, $\nu$ is the kinematic viscosity and $\alpha$ is a concentration-dependent parameter

$$\alpha = F(C) \quad (3)$$

The values of $V_{\text{crit}}^*$ and $\alpha$ are properties of the polymer solutions, and are usually determined from pressure drop measurements in pipes. For dilute solutions, $V_{\text{crit}}^*$ appears to be independent of the polymer concentration, whereas $\alpha$ is proportional to the concentration. At moderate concentrations $\alpha$ increases only slightly with the concentration.[6,7] The maximum values of $\Delta B$ measured are of the order of 28.

Meyer's equation implies, of course, that the structure of the turbulent flow outside the sublayer and the buffer zone is unaffected by polymer additives. This conclusion has been supported by measurements with several types of polymers,[4,6] however, some observations suggest that it might not be universal, and in certain cases some changes in the structure of the larger eddies also occur.[3,7] These changes, as well as many other non-Newtonian phenomena observed in polymer solutions, were usually observed in higher concentrations than those necessary for obtaining a large drag reduction. Several experiments also indicate that when a solution is diluted or stirred for a long period, these phenomena disappear without affecting the ability of the solution to reduce the drag. It has therefore been suggested that the non-Newtonian behavior and effect on the large eddies are produced by molecular cross-linking or micro-entanglements which are of no value or disappear when the solution is diluted and are not necessarily related to the drag reduction.[3,8,9] In this work we shall limit ourselves to polymer solutions which do not affect the large eddies, and describe the increase in the thickness of the sublayer by Meyer's equation.

Since drag reduction is affected through changes in the structure of the sublayer, it is natural to assume that in the case of nonuniform concentration fields, it is the concentration at the wall which determines the drag reduction. We shall accept this assumption since we shall address ourselves to high-speed flows in which thickness of the viscous sublayer is small and the concentrations within the layer do not vary considerably. In many other cases, the thickness of the viscous sublayer might be large, and if the polymers are ejected within the sublayer, large gradients could exist normal to the wall in the region $5 < zV^*/\nu < 30$. It is exactly in this region where the effect of the polymers is noticed, and if the concentration here is not constant, there is at present no way to predict its effect on the basis of measurements in homogeneous solutions.

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Received April 5, 1971. This work is based in part on the M.S. thesis of the second author at the University of Iowa. The work has been sponsored by the Office of Naval Research, Contract Nonr-1611(03).

Index categories: Hydrodynamics; Boundary Layer and Convective Heat Transfer–Turbulent.

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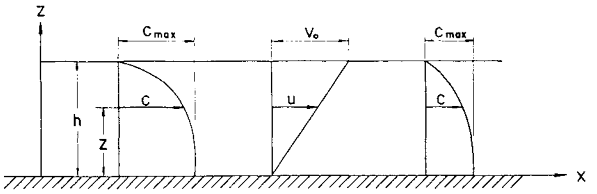

**Fig. 1 A model of molecular diffusion from a laminar layer.**

It should also be noted that the concentration of polymers downstream from a source is time dependent. The observed nonlinear dependence of drag reduction on the concentration suggests that homogeneous solutions should be more effective than nonhomogeneous solutions with the same mean concentration. On the other hand it was reported by Kovalsky[10] that the magnitude of drag reduction was hardly changed when the average flux of the ejected polymers was drastically reduced by stopping the injection periodically for very short durations. These surprising results were attributed to the effect of an absorbed layer of polymers on the wall which has a significant role in drag-reduction phenomena. Similar observations were reported by Little.[11] In the absence of an accepted theory of drag reduction, we shall assume that drag-reduction in time-dependent concentration fields is related to the mean concentration as in uniform solutions.

Poreh and Cermak[1] who studied the diffusion of tracers from a line source near a wall distinguished four stages of diffusion in a turbulent boundary layer:

1) An initial stage close to the source, in which a large portion of the diffusion boundary layer is submerged in the viscous sublayer. The length of this region is determined by the initial conditions near the source, physical size of the source relative to the thickness of the sublayer, the ejection velocity and the magnitude of the molecular diffusivity.

2) An intermediate stage, in which the diffusion boundary layer is submerged in the momentum boundary layer and its thickness is large compared to that of the sublayer. The diffusion rate in this region is relatively large, and the concentration profiles are found to be approximately similar in the sense that

$$c/C_{\max} = f(z/\lambda) \tag{4}$$

where $\lambda$ is a characteristic size of the diffusion boundary layer defined as the distance from the wall where $c/C_{\max} = 0.5$. Measurements in air indicate that the intermediate stage extends 20–40 boundary-layer thickness downstream from the source (see Fig. 3).

3) A transition stage, with a somewhat slower diffusion rate due to the lower level of turbulence in the outer portion of the boundary layer.

4) A final stage, in which the growth of the diffusion boundary layer coincides with that of the momentum boundary layer. In this stage the maximum concentration, near the wall, is inversely proportional to the thickness of the boundary layer and the ambient velocity,[1]

$$C_{\max} = Q/0.55\delta U \tag{5}$$

## Diffusion of Polymers in the Initial and in the Final Zones

The molecular diffusivity $D$ of high molecular weight polymers is expected to be very small. For example, the value of $D$ for WSR-301 at infinite dilution is estimated, using the Mandelker and Flory model,[12] to be smaller than $10^{-12}$ m$^2$/sec, suggesting that polymers emitted at the wall would hardly diffuse over fairly large distances downstream.

Consider, for example, a viscous layer of thickness $h$, as described in Fig. 1. Let the diffusivity be represented by $D$, the velocity by $u = V_0 z/h$, and let the concentration of polymers, to be denoted by $C$, at the upper edge of the layer be given by $C(h) = 0$. The decay of the maximum concentration $C_{\max}$ at the wall in this case is given by[13]

$$C_{\max}(x) = C_0 e^{-x/L_0} \tag{6}$$

$$L_0 \simeq 0.1\, v_0 h^2/D \tag{7}$$

The length $L_0$ indicates the distance at which the maximum concentration drops to $e^{-1}$ of its original value. Equation 7 can also be written as

$$L_0/h \simeq 0.1(v_0 h/\nu)(\nu/D) \tag{8}$$

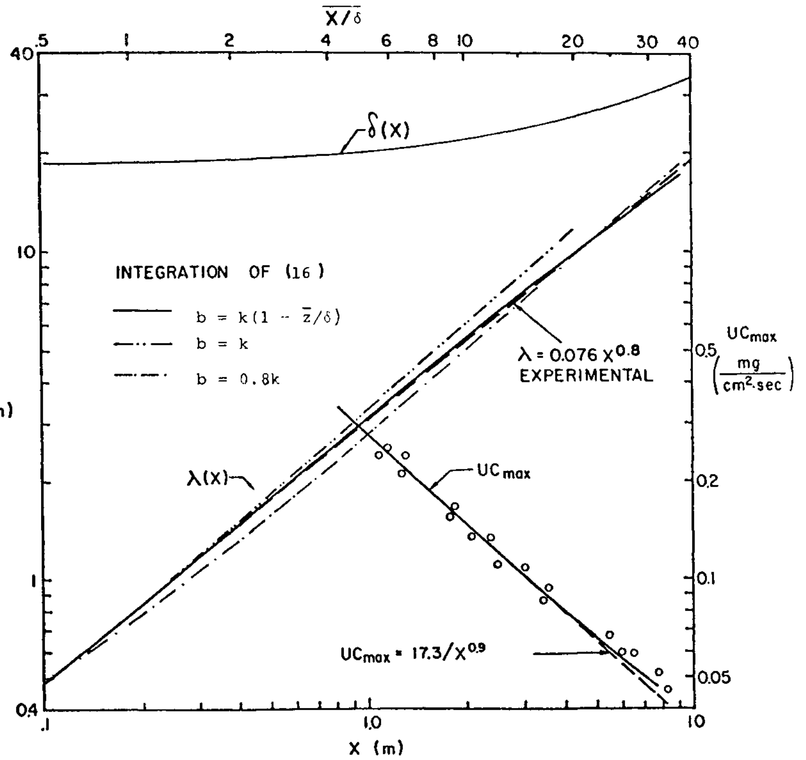

**Fig. 2 Diffusion of inert scalar from a line source.**

Letting $hV^*/\nu = 5$ and $V_0/V^* = 5$, as in a viscous sublayer, one finds that

$$L_0/h \simeq 2.5(\nu/D) \tag{9}$$

Thus, for $\nu/D = 0(10)^6$ and $h = 0.1$ mm, $L_0$ is of the order of 25 m, indicating a very slow rate of diffusion. The diffusion of the polymers from a real viscous sublayer is, however, many times stronger. Although the mean velocity profile in the sublayer hardly deviates from the laminar velocity distribution, $u/V^* = zV^*/\nu$, the turbulent fluctuations penetrate deep into the sublayer and increase the diffusion rate. Hanrathy[14] suggested that the effective value of $\epsilon/\nu$, where $\epsilon$ is the turbulent diffusivity, in the region $zV^*/\nu < 2.5$ is given by

$$\epsilon/\nu = 0.00032(zV^*/\nu)^4 \tag{10}$$

Accordingly, $\nu/\epsilon$ at $zV^*/\nu = 2.5$ would be of the order of $10^2$. Replacing $\nu/D$ in Eq. (6) by $\nu/\epsilon$ and assuming that it is of the order of $10^2$, one finds that $L_0$ is drastically reduced.

Since viscous drag increases rapidly with speed, drag reduction is particularly attractive in the case of fast moving bodies. The thickness of sublayers in flat plates and pipes can be estimated from[15]

$$hV^*/\nu = 5 \tag{11}$$

and

$$(V^*/U) = 0.17(UL/\nu)^{-1/10} \tag{12}$$

It follows from these equations that in most practical cases $h$ would be smaller than 0.1 mm. Even if the polymers double or triple the thickness of the sublayer, it still will remain very small, and $L_0$ would be only of the order of a few centimeters. The surface roughness and the disturbance at the ejection point will further increase the rate of diffusion from the sublayer. One may therefore conclude that except in very slowly moving ships and in ship models scaled by the Froude number, the initial stage of diffusion would be relatively short and insignificant.[16]

In the final stage of diffusion, on the other hand, there are hardly any polymers left in the viscous sublayer. Since the structure of the major portion of the turbulent boundary layer is not affected by the polymers, their distribution in the final stage is expected to be similar to the distribution of inert tracers. Measurements of polymer concentrations in that zone reported by Wetzel and Ripken[17] confirm this conclusion. The same measurements have also been analyzed by Fabula and Burns[18] who found reasonable agreement with the data of Poreh and Cermak[1] in the final zone, but recommended a change of about 9% in the value of the constant in Eq. (5). Thus the main change between the diffusion of polymers and other tracers would be in the intermediate zone.

## Diffusion of Polymers in the Intermediate Zone

Two approximate methods have been used in the past to predict gross diffusion patterns in turbulent flows. The most widely known method employs the eddy diffusivity model which assumes that the flux of the diffused matter by the turbulent fluctuations is proportional to an eddy diffusivity times the local concentration gradient,

$$q_z = -\epsilon\ (\partial c/\partial z) \tag{13}$$

In introducing such a model it is assumed that $\epsilon$ is a function of the flowfield and that its value at a point can be specified regardless of the position of the source. Study of diffusion in homogeneous turbulence has revealed that $\epsilon$ is independent of the position of the source only at distances from the source that are large compared to the Lagrangian integral scale of the turbulence. Measurements of the diffusion pattern in boundary layers[1] indicate that the same limitation holds in the case of diffusion in turbulent shear flows, and that $\epsilon$ cannot be related to the local characteristics of the turbulent motion downstream from a line source as long as the distance to the source is smaller than the Lagrangian integral scale of the boundary layer. Estimates of the Lagrangian integral scale suggest that it is of the order of 10 boundary-layer thickness, which means that the use of an eddy diffusivity model in the intermediate zone is not fully justified.

Batchelor[19] introduced a Lagrangian similarity hypothesis to predict the mean turbulent motion of particles in steady, self-preserving shear flows. The Lagrangian similarity hypothesis yields the following equations describing the change of the mean verticle position $\bar{z}$ and the mean longitudinal position $\bar{x}$ for an ensemble of single-particle releases in the logarithmic portion of a boundary layer[20]

$$d\bar{z}/dt = bV^* \tag{14}$$

where $b$ is a constant, termed Batchelor's constant, and

$$d\bar{x}/dt = u(\bar{z}) \tag{15}$$

It follows that

$$d\bar{z}/d\bar{x} = bV^*/u(\bar{z}) \tag{16}$$

If the probability density function of the ensemble, which is expected to be a universal of $(x - \bar{x})/\bar{z}$ and $(z - \bar{z})/\bar{z}$, is assumed to have a sharp maximum at $x = \bar{x}$, one may further conclude[20] that the boundary concentration downstream from a continuous line source is given by

$$C_{\max} \propto Q/\bar{z}u(\bar{z}) \tag{17}$$

where $Q$ is the discharge of the source (per unit width).

Ellison[21] has estimated that Batchelor's constant is given by $b = k$, where $k$ is the von Kármán constant [$1/k = A$ in Eq. (1)]. His analysis suggests that $\bar{Z}$, the mean position of particles at a given cross section $x$, is equal to $\bar{z}$, the mean position of a single particle releases when $\bar{x} = x$. Ellison's estimate has been supported by Pasquill[22] who compared a few field observations of the mean position $\bar{Z}$ and found them to be virtually identical with the theoretical values obtained by integration of Eq. (16) with $b = k$.

The assumption that

$$\bar{z} \simeq Z \tag{18}$$

has been further supported in a previous analysis of the authors[23] who have shown that the measurements of Poreh and Cermak[1] in the intermediate zone satisfy an equation similar to Eq. (17),

$$C_{\max} = Q/1.45\bar{Z}\mathrm{u}(\bar{Z}) \tag{19}$$

where, applying Eq. (4) and the experimental values of $c$ from[1]

$$\begin{aligned}\bar{Z} &= \int_0^\infty czdz/\int_0^\infty cdz\\ &= \lambda \int_0^\infty f(\xi)\xi d\xi/\int_0^\infty f(\xi)d\xi \simeq 0.76\ \lambda\end{aligned} \tag{20}$$

Figure 2 from Ref. 23 compares the experimental data with numerical integrations of Eq. (16) using $b = k$, $b = 0.8k$ and $b = (1 - \bar{z}/\delta)$ ($x$ in Figs. 2 and 3 is the distance from the source). The theoretical calculation with $b = k$ coincides with the experimental curve, $\lambda = 0.076\ x^{0.8}$, when the diffusion boundary layer is completely submerged in the logarithmic layer ($z/\delta < 0.15$). When the tracer leaves this layer, however, its mean upward velocity, Eq. (14), decreases. This is probably due to the decrease of the turbulent fluctuations. As shown in Figs. 2 and 3, the integration of Eq. (16)

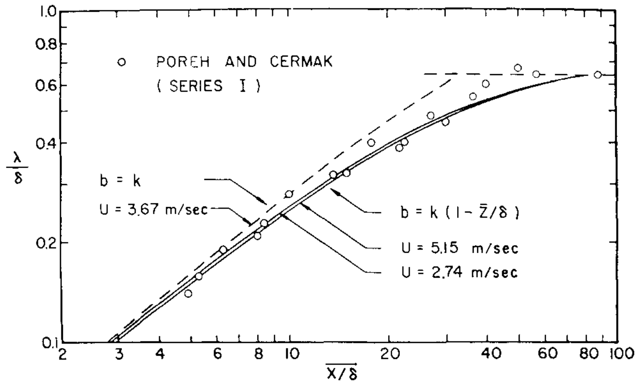

**Fig. 3 Growth of the diffusion boundary layer within the momentum boundary layer.**

with

$$b = k(1 - \bar{z}/\delta) \tag{21}$$

gives reasonable agreement with the data throughout and beyond the intermediate zone. Figure 3 compares the change of $\lambda/\delta$ vs $\overline{x/\delta}$, where $x$ is the distance from the source and

$$\overline{x/\delta} = \int_0^x \delta^{-1}(x)dx \tag{22}$$

with the measurements of Poreh and Cermak. The inhomogeneity of the boundary layer was taken into consideration by using at each section the local values of $V^*$ and $\delta$. The local velocity $u$ was calculated from the outer laws:

$$(U - u)/V^* = F(z/\delta) = K(1 - z/\delta)^2 \qquad z/\delta > 0.15 \tag{23}$$

and

$$u/V^* = U/V^* - B_2 + A\log(z/\delta) \qquad z/\delta < 0.15 \tag{24}$$

The development of the boundary layer was calculated using Ladweber's solution.[24]

An estimate of the diffusion rate of drag reducing polymers and of the drag reduction in the intermediate zone can be made by assuming that the laws describing the velocity profile in the log and outer regions in case of homogeneous solutions [Eqs. (1, 23 and 24)], as well as the empirical laws describing the diffusion [Eqs. (16–21)], apply to this case as well. The use of Eq. (16) is consistent with the use of Meyer's equation. Equation (1) and all other measurements which find that the polymers do not affect the velocity defect law imply that "the polymers do not significantly affect the eddy viscosity."[4] The similarity between momentum and mass transport suggests that the transport of the polymer by eddies, which are many fold larger than their size, would not be affected either. It should be realized, however, that the use of either set of equations could provide only estimates of the drag and diffusion very close to the source, as it is clear that the entire momentum boundary layer will not respond immediately to rapidly varying boundary conditions at the wall. The effect of the injection of polymers would be similar to that of a sudden decrease in roughness. Measurements of the local shear after a decrease in roughness[25] reveal that the surface shear and the flow very close to the wall respond immediately to the boundary condition and even slightly overshoot the new equilibrium values. On the other hand the flow away from the wall changes gradually toward the new equilibrium state. Thus, the proposed method of calculation slightly overestimates the diffusion in the inner region but underestimates the diffusion rate in the outer region, but since the diffusion boundary layer coincides, more or less, with that region near the wall which has sufficient time to respond to the boundary conditions, the error is not expected to be large.

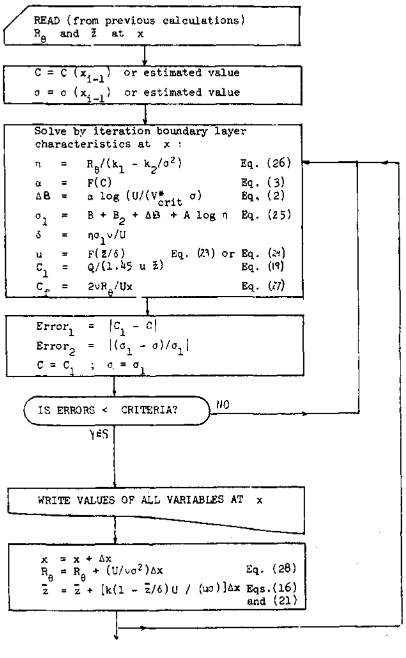

**Fig. 4 Schematic flow diagram for computer program.**

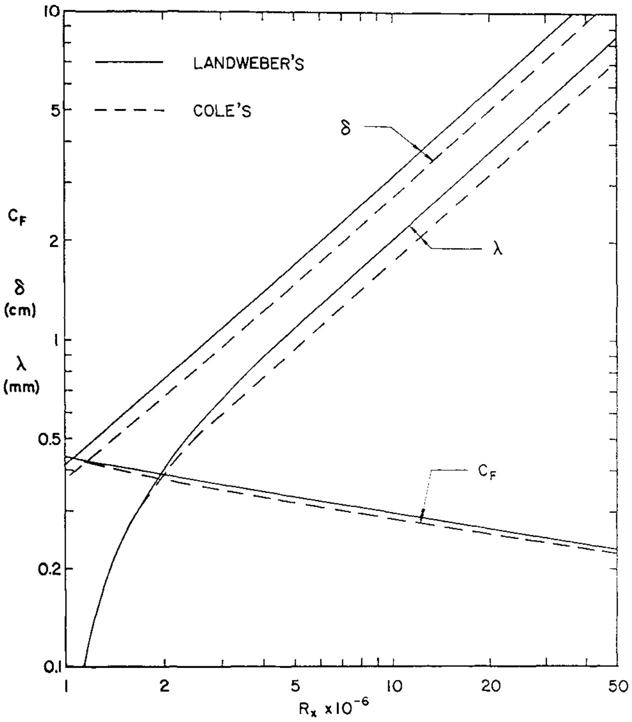

**Fig. 5 Numerical integration of Eqs. (16) and (20) using Landweber's and Cole's coefficients.**

The requirement that both Eq. (1) and Eq. (24) describe the velocity profile in the logarithmic region gives

$$\sigma = A\ln\eta + B_1 + B_2 + \Delta B \tag{25}$$

where $\sigma = U/V^*$ and $\eta = \delta V^*/\nu$. The momentum thickness Reynolds number, $R_\theta$ in this case is given by[24]

$$R_\theta = \eta(k_1 - k_2/\sigma) - k_3 - k_4/\sigma \tag{26}$$

where

$$R_\theta = \frac{U}{\nu}\int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \tag{27}$$

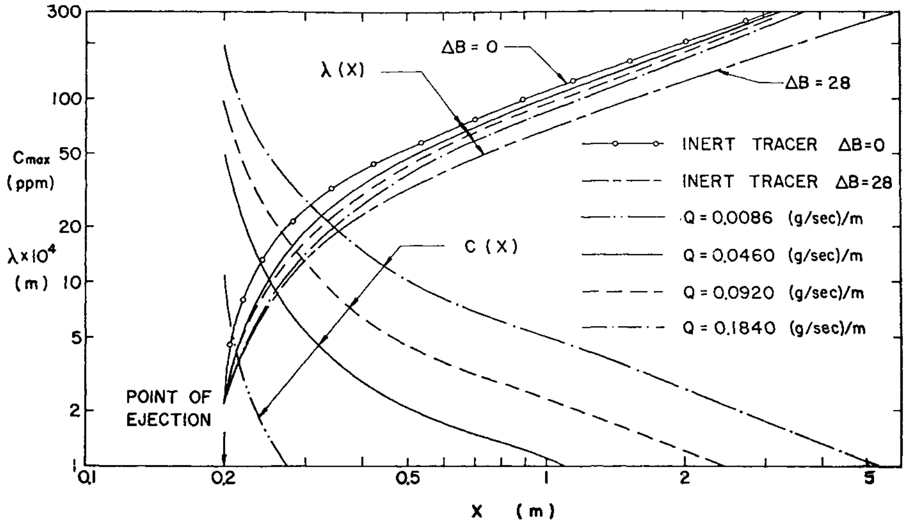

**Fig. 6 Diffusion of drag reducing polymers in a developing boundary layer. ($U$ = 5 m/sec).**

and $k_1$-$k_4$ are constants. The values of these constants depend on the form of the velocity profile. The contributions of $k_3$ and $k_4$ are usually negligible at high Reynolds numbers, and they are identically equal to zero if the contribution of the sublayer to $R_\theta$ is neglected. The coefficients $k_1$ and $k_2$ are in this case universal constants independent of $\Delta B$.

Landweber[24] has used the values $A = 2.606$, $B_1 = 4.0$, $B_2 = 2.0$, and found that $k_1 = 3.5$, $k_2 = 23.3$, $k_3 = 33$ and $k_4 = -435$. Coles[26] has neglected the contribution of the sublayer and recommended the use of $A = 2.5$, $B_1 = 5.1$, $B_2 = 2.8$, $k_1 = 4.05$ and $k_2 = 29$. Slightly different values were recommended by Granville,[27] who took into consideration both the sublayer and a transition between the sublayer and the logarithmic region.

The shear stress at $z = 0$ for zero pressure gradient boundary layers is always given by the change of the momentum flux, thus,

$$dR_\theta = dR_x/\sigma^2 \qquad (28)$$

or, where $R_x$ is the Reynolds number,

$$R_x = \int \sigma^2 dR_\theta \qquad (29)$$

Equation (29) can be integrated analytically when $\Delta B$ is a constant.[24] An analytical solution has also been presented by Granville[2] for the final stage of diffusion where $C_{\max} \propto Q/(\delta U)$. In the intermediate zone where $C_{\max}$ is determined by the diffusion of the polymers, the diffusion rate and the boundary layer development can be solved by simultaneous numerical integration of Eq. (16) and Eq. (28). A flow diagram of the essential features of a computer program for solving this problem is outlined in Fig. 4. Figure 4 shows the equations used in the intermediate zone. When $\lambda$ reached the value 0.64, Eq. 19 in the program was replaced by Eq. (5) making it possible to continue the numerical computation of the boundary-layer development in the final stage of diffusion.

To demonstrate the role of the intermediate zone and the effect of the drag-reducing properties of the polymers on the diffusion, a few examples have been calculated. The calculations were performed using a predictor-corrector method based on the flow diagram presented in Fig. 4. The values of the variables at $x_i + \Delta x$ were first estimated and then calculated using the average of the values of $x_i$ and the estimated values at $x_i + \Delta x$. The value of $\Delta x$ used in the numerical integration was smaller than $\delta/4$ near the source and equal to $\delta$ at large distances. Further reduction of $\Delta x$ did not affect the results significantly. The results of the numerical calculations are summarized in Figs. 5–9.

Numerical solutions of the boundary-layer development, the drag coefficient $C_F = D/\rho U^2 x$, and the diffusion of an inert tracer, calculated with Landweber's and Cole's coefficients are compared in Fig. 5. The polymers were ejected at $X = 0.2$ m from the leading edge and the ambient velocity was taken to be $U = 5$ m/sec. The drag coefficient at the ejection point, $R_x = 10^6$, was assumed to be $C_f = 0.0044$ and the initial value of $\lambda$ at this point was assumed to be 0.22 mm.

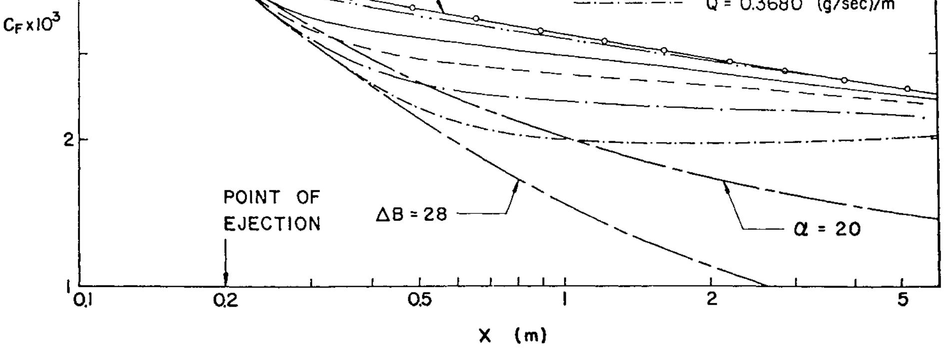

**Fig. 7 Drag reduction by ejection of polymers near the wall.**

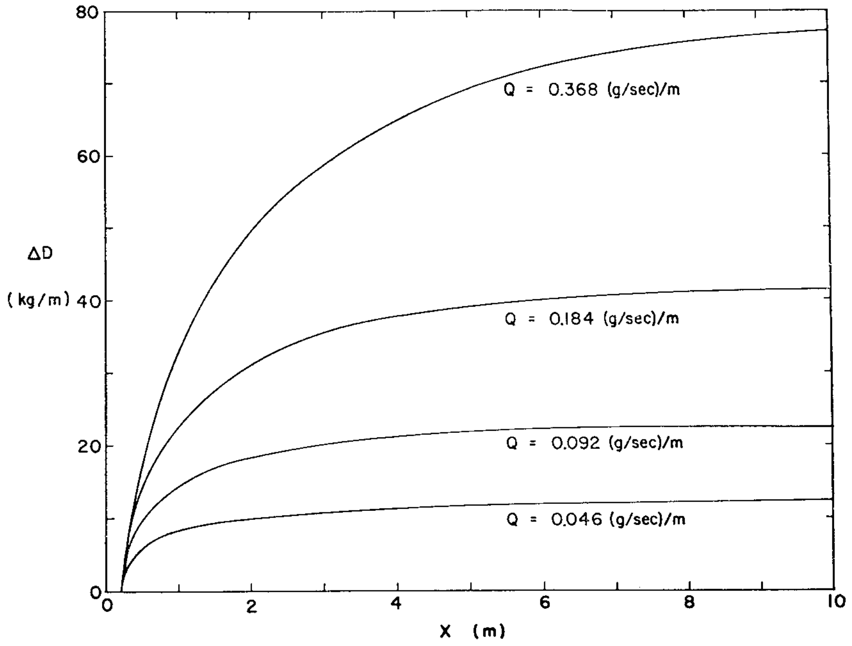

**Fig. 8 Reduced drag with polymer ejection as a function of the length of of the plate.**

The results of the two numerical integrations were also compared with the analytical solution of Eq. (26) by Landweber and Cole and were found to be practically identical. The values of $\delta$ calculated with Landweber's coefficients are, as shown in Fig. 5, approximately 10% larger than those calculated with Cole's coefficients. The asymptotic values of $\lambda$ differ, of course, by the same amount, as it was assumed that in the final zone $\lambda = 0.64\delta$. The development of the diffusion boundary layer in the intermediate zone is, however, practically identical in both calculations. The difference between the values of the drag coefficient is also found to be small. In the rest of the integrations we have used only Landweber's coefficients.

The diffusion of drag reducing polymers ejected at $X = 0.2$ m in the same flow is shown in Fig. 6. A linear relationship, $\alpha = 1.0C$, where $C_{\max}$ is the concentration in ppm was used to approximate Eq. (3) up to $\Delta B = 28$. This value was assumed to be the maximum possible shift of the logarithmic profile, giving

$$\Delta B = C \log(V^*/V^*_{\text{crit}}) \quad \text{for } 0 < C_{\max} \log(V^*/V_{\text{crit}}) \leq 28$$

$$\text{and} \qquad (30)$$

$$\Delta B = 28 \quad \text{for } C \log(V^*/V^*_{\text{crit}}) > 28$$

The effect of the drag-reducing properties of the polymers on the diffusion in the boundary layer is clearly seen in Fig. 6. The curve $\Delta B = 0$ shows the diffusion of an inert tracer in a boundary layer without polymers. The curve $\Delta B = 28$ shows the diffusion when the drag reduction is maximum. One sees that the diffusion rate decreases in the presence of drag reduction. When drag reducing polymers are ejected near the wall at $X = 0.2$ m the diffusion rate is determined, as indicated by Eq. (16), by $V^*/u(\bar{z})$ or approximately $V^*/U$, which is a function of the concentration near the wall. When the discharge of the polymers is small, the diffusion boundary layer develops almost as in case of inert tracers, i.e., $\Delta B = 0$. When $Q$ is large the diffusion follows initially the curve $\Delta B = 28$; however, further downstream the concentration decreases and the diffusion rate increases.

Figure 7 shows the values of the drag coefficient, $C_F$, as a function of $X$, the distance from the leading edge. The curve for $\Delta B = 28$ reveals significant drag reduction for long plates. However, to maintain such a large value of $\Delta B$ it would be necessary to have a very high concentration of polymers along the entire plate. The curve $\alpha = 20$ shows the drag reduction for a uniform ($C = 20$ ppm), starting at $X = 0.2$ m. Drag reduction of slightly more than 40% is obtained, in this case, for plates 1–5 m long.

Drag reduction with polymer ejection depends, of course, on the amount of polymers ejected at the source. When $Q$ is large, the initial drag reduction approaches the curve $\Delta B = 28$. One sees from Fig. 7 that the longer the plate, the larger are the quantities required to achieve the same percentage of drag reduction. The absolute power saving due to the polymer ejection does not decrease, however, for longer plates, as is demonstrated in Fig. 8, where $\Delta D$, the difference in the drag with and without polymers, is plotted vs $X$. One can also see from Fig. 8 that the drag reduction per unit discharge, $(\Delta D/Q)$, is larger for small values of $Q$, indicating that it might be more economical to obtain only small drag reductions.

The numerical integrations indicate that the length of the intermediate zone in the presence of drag reducing polymers is increased only slightly. In the case of $Q = 0.368$ (g/sec)/m, for instance, $\lambda$ reaches its asymptotic value $0.64\delta$ at $X = 0.98$ m ($\overline{x/\delta} = 82$). In spite of its relatively short length, the effect of the intermediate zone on the drag reduction is large. Figure 9 presents a comparison of the values of the drag reduction calculated earlier with the values of the drag reduction obtained with the same polymer discharge assuming in the latter case that due to improper injection technique, the polymer diffuse immediately throughout the boundary layer and the wall concentrations are determined by Eq. (5) rather than Eq. (19). The results demonstrate that the additional drag reduction achieved by injection near the wall is of large magnitude although the intermediate zone is short. This is due to the relatively large shear stresses in this zone and the effect of the reduction in the drag in the intermediate zone on the value of $\delta$ in the final zone. The smaller the upstream drag, the smaller the boundary-layer thickness $\delta$, which gives, according to Eq. (5), higher concentrations near the wall. For very long plates, however, the effect of intermediate region is negligible.

## Conclusions

The diffusion of diluted drag reducing polymers has been considered. A numerical method for calculating the diffusion and the drag reduction in case of polymer ejection from the wall has been outlined and a few examples have been cal-

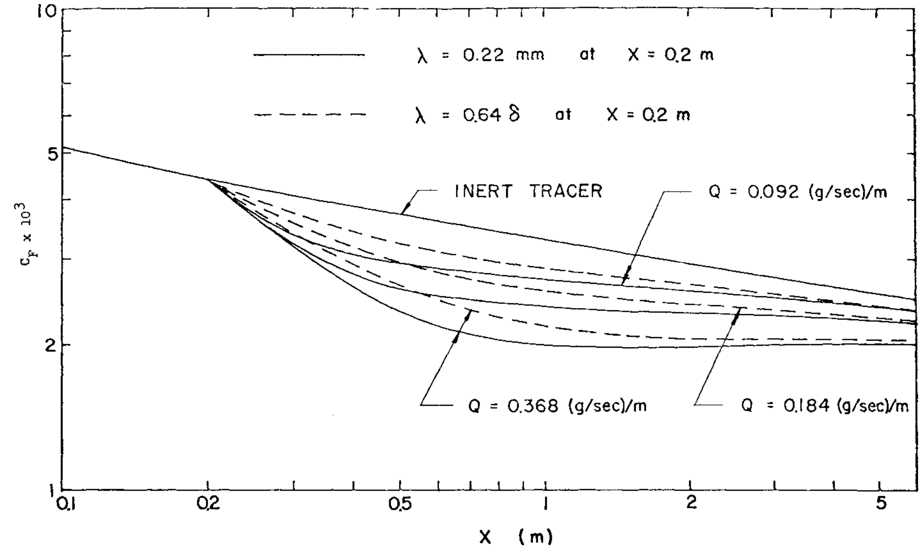

**Fig. 9 Effect of initial conditions on drag reduction.**

culated. Drag reduction in dilute polymer is affected through an increase of the viscous sublayer. In high-speed flows, however, the effect of the viscous sublayer on the diffusion is limited to a short distance near the ejection point. Further downstream, diffusion is similar with and without polymers. In both cases the diffusion rate is proportional to the shear velocity, and inversely proportional to the mean velocity. Thus, the diffusion rate is decreased in case of drag reduction. The effect, however, is not very large and the polymers "fill" the entire boundary layer within a distance smaller than 100 times boundary-layer thickness. Still, it might be significant in case of short plates.

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